Chapter 5

"You are my fourth math teacher; how can you teach me math?"

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Abstract

In this essay, I will tell a story that started with a comment from a former pupil I had in high school in mathematics. His comment made me reflect on my own practice as a math teacher. Even with a lot of experience, it is possible that this may obscure new ways of teaching. If we can be more flexible when planning our teaching and lectures, there is a greater chance to reach even more pupils. When teaching at the university, I have this in mind, where the focus is on an active student and myself as a reflective teacher.

Keywords

Self-efficacy, flexibility, to see the individual, motivation.

Introduction

As a former pre-high and high school teacher and now a pre-school teacher at the university, I will tell a story from high school in this essay. I will use philosopher Anders Lindseth's three steps in reflective practice research as a guideline. By using this framework, it helps me reflect on my own practice and learn to develop myself as a researcher. For further details about the method, I will recommend reading chapter 1 in this book. By telling my story, I hope this will help other future preschool teachers reflect on their own praxis. Hopefully, with that result, even more pupils will succeed in mathematics.

In the first part of the essay, I start with my concrete reflection; thereafter, I will move on to the critical reflection and, finally, the theoretical reflection.

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In the theoretical reflection, I discuss the concept of self-efficacy developed by the psychologist Albert Bandura. According to him, self-efficacy plays a major role in shaping human behaviour, motivation, and achievement. The importance of facilitating dialogue and asking open questions, like in the Socratic dialogue, will also be discussed.

To facilitate communication among students, it is essential that we be flexible when organizing our lectures. Furthermore, if we manage to differentiate classes into smaller groups, it could be easier to follow up with students. For instance, when I am using vertical surfaces, the students must take a more active role in their learning when discussing different solutions and strategies for problem solving. To let the students be even more prepared for their practice, I have organised different types of simulations where different topics have been lifted. For instance, feedback from this form of organising my lectures has given me fruitful insight and made me even more reflective on my own praxis.

Concrete reflection

"You are my fourth math teacher; how can you teach me math?", was a comment I got from a pupil four weeks after I started up with a new class in a new year at high school. Even though I had many years of experience, I felt this comment block my brain for a "few seconds", while my thoughts raced over my role as a mathematician teacher. I am not sure, but I think he saw my facial reactions, such as blushing, as I fumbled with my whiteboard pencils. Although I had been teaching similar classes in the same field, I had never heard such a comment. My former experience was such that a major part of the class had low confidence and some were bored with mathematics, but I thought I had reached them with my enthusiasm for mathematics.

So even with my experience, in a period of four weeks, I had not convinced him that he could perform and learn better mathematics. I implemented many things from my teaching practice. I had done repetition on the whiteboard before starting up with new themes. I let them work alone, in pairs, and in a little bit of a flipped classroom. So even with my excited and active role as a mathematics teacher, I had not hit a nerve or triggered any form of understanding in him so far.

With my teaching experience, I thought I had gained a thick skin. But his comment showed me that my skin wasn't so thick after all. His comment struck me more than other comments from previous pupils. I heard myself thinking, "Yes, you are his fourth teacher, and you are not so special".

After the school day ended, I went home still thinking about my teaching methods and practice. I started to reflect on my practice and thought that my teaching was not so special. I was just an ordinary brick among other teachers. He did not say that I was a poor math teacher, but he said it in a manner that made me feel that it was my fault for not reaching him and not being able to teach him mathematics. I wondered if this comment would be made in sports. For instance, "teach me how to be a good swimmer", or "teach me to be a good football player". It was strange, and I thought that after all my years in the school, one comment from a pupil would make me reflect on whether my teaching was good enough.

At the next lecture, I found it natural to chat with him and talk about his experience with my mathematics lectures. I soon recognized that he was not so comfortable in the beginning, but I must admit it was exciting for me also, although I had tried to make the meeting and conversation unformal. I have arranged many meetings through my years as a math teacher, but I thought this was different because I felt the need to show him that I am better than his impression of me.

As the conversation moved on, it became clear to me that he had not participated in so many conversations with his former math teachers. I asked him if he was challenged in mathematics and how often. Not so often, he answered. My first thought was that I wanted him to talk about mathematics, challenge him, and set demands.

One of the first things I did was organize my way of lecturing. Not only because of him but for everybody in the class. Instead of standing alone at the whiteboard, I organized the whole class into groups and gave tasks on five different topics with increasing difficulty and progression. Many of the tasks were relatively easy to solve, but that was conscious. After a week, I felt the atmosphere in the classroom get warmer. Someone would say that this is obvious since they were getting to know each other better, but I felt most of the pupils took part in the warmness.

One day, when we were talking about the tasks, the pupil said to me: "I don't have a math brain". I thought it was surprising and nice that he wanted to talk about his feelings for mathematics. I asked him why he felt so, and he answered, "Because nobody in my family has been clever in math". My immediate response was, "So that means you can't be clever in mathematics?" He looked surprised and said "No". I continued, "You have shown me and your group that you are capable of solving tasks at a medium level, that you have taken an active part in your learning, and that you have participated in the group discussion". I asked him if he thought it had influenced him that the other family members weren't so clever in mathematics, and the answer was "maybe". I thought that perhaps some

of his low confidence in mathematics and a passive role were triggered by a "self-fulfilling prophet" for him. I asked why he managed a high grade in gymnastics, and he said, "Because I like it". More and more, it became obvious to me that his parents, his surroundings, and himself combined to create low self-confidence and a passive attitude toward learning in him.

After approximately three months, I felt I had gotten in better touch with him. He seemed to be more interested in understanding his missing parts, for instance, the summation of fractions. By involving me in what he wanted to improve, I gave him and his group appropriate tasks. I did not push him to start up with the syllabus and placed that on hold. Maybe it was my practical wisdom that decided it had to wait. This meant that he and I had to find his lack of knowledge before we moved into more advanced mathematics.

I felt that I had a nice dialogue with everyone in the class now, and many of them improved at solving problems and explaining the different solutions to each other. My pupil in my story moved further and further away from the stage of just remembering toward the stage of understanding. In this period, he was funny and smiled when he managed to explain.

One day, I asked what his plans were for the rest of the year and if he had thought about what he wanted to work on. I said I had suggestions for helping him overcome the syllabus and said, "You need to take a bit more active part in your plans". We need to stake out a plan that will help you through the syllabus. "I will", he answered.

After approximately four months, I arranged a formal written test with a number of tasks from the syllabus. I must admit I was nervous, but I knew he had raised his mathematical understanding. This was also a test for me. I hoped I had managed to capture his interest in mathematics and motivate him to take a more active role in his own learning. I became relieved. His results were roughly above average. I will never forget the smile on his face. When he saw the mark, he beamed happily and thanked me for seeing and helping him. I am not sure if I saw a tear in his eye, but I became emotional. During the rest of the school year, I was still focusing on what we had staked out.

The whiteboard was, of course, still in use, but to a much lesser extent. As the months progressed, I still had questions and challenged him in areas within the syllabus. He was making progress. At the end of the school year, we arranged a common written test for all pupils in the same field of study. He had made a great leap forward, and he thanked me a lot for my remarkable help. One week before the school year ended, I got a phone call. A voice said, Thank you for your patience with our boy; we now have a member in our family that can do math.

Critical reflection

With over 15 years of experience in junior high and high school, the few words from the pupil made me reflect even more on my teaching practice.

There could be many reasons why he said those words, but in the following, I will figure out the topics that are at stake in this story.

Experience may obscure seeing the pupil

Even if I have a lot of experience and have been educated in pedagogy and mathematical didactics, that does not mean that my register is fully completed. Of course, I have received feedback from my pupils, their parents, and the results of the exam. However, I am not sure that this is synonymous with my lectures and sessions being good all these years. Of course, I could improve my methods of teaching. Since all teachers get feedback, many reflect on their own praxis, but I am not sure if it is inner or outer motivation that makes them reflect on their own praxis.

If I am using almost the same examples year after year and doing my own practice without extensive cooperation with other teachers and at other schools, the possibilities are great that my form of teaching will not be updated, and I am on my own track.

It is of great importance that we keep in mind that classes consist of individuals with different social and academic backgrounds. Our lessons need to take this into consideration, and we should do our best to be flexible according to plans and assessments.

To really see the pupil, we need to make allowances accordingly by changing plans and adjusting the curriculum until the pupil has understood the basic parts. We don't need to rush with written tests that just confirm the poor results from the last written test. My own experience is that some of these tests end up in the garbage without my notes having been read or at least understood.

To see the pupil, we also must set demands. There are several instances in my career where my manner of communication was too nice and passive, causing the pupils to think it was the same for me whether they had done their homework or not. When thinking back, perhaps I had been too kind and affectionate when the pupils did not do their homework.

Since the class consists of individuals, we should facilitate opportunities to give them choices to let them show their mathematical competence in different ways.

Theoretical reflection

In my years as a contact teacher, I have heard parents say: "I don't have the ability for math" or "I don't have a math brain", as their child is sitting

beside them. But I have never heard them say: "I don't have the ability to learn English" or "I don't have the ability to learn social studies". These comments have made me wonder many times when talking about their child's progression in mathematics. It is strange that some people might actually believe that weak results or missing parts of understanding in mathematics are hereditary. I am sure that every parent always wants the best out of schooling for their child.

I still believe that some parents have a great understanding that their child is performing weaker in mathematics relative to other topics at the school. Maybe it is because they have had the same experience with mathematics. In the same meetings, I have heard parents say that topics like history and social science are typical "memorizing topics", and the ability to get better marks is much easier. In these meetings, I have tried my best to encourage the pupils and argue that it is possible to perform at a high level in mathematics if they want. I don't think that my experience is unique. I believe that there are many similar stories where low self-efficacy is the result.

Building up self-efficacy

My pupil's belief in his ability to successfully complete a task or achieve a specific goal was low in the beginning. I had to motivate him. I had to gain knowledge and found a very interesting article by the Canadian Psychologist Albert Bandura. According to Bandura (1994, 1997), self-efficacy plays a major part in shaping a human's behaviour, motivation, and achievement, where low self-efficacy can lead to feelings of helplessness, frustration, low motivation, and giving up easily when faced with obstacles and setbacks. This was a new start for me.

Since I had to see my pupil do better and let him grow into believing in himself, I also had to better organise my teaching. I had to let the class show different ways of demonstrating mathematical competence and make my mathematical tasks better. Moreover, I had to focus on what Sullivan et al. (2015) argue: that tasks with multiple ways to solve the problem may trigger enthusiasm and motivation. By using other strategies they are familiar with, the students develop and gain a better understanding.

Stillman et al. (2009) explain that when students are solving challenging tasks, they are on the edge of their comfort zone. These types of challenges may give the students the experience that the right solution is not easy to find, like real problems, which are often complicated. It was challenging to find appropriate tasks that could trigger him and the rest of the class. One of my solutions was to open up mathematical tasks. Sullivan et al. (2015) argue that tasks that can be solved in different ways demand an open structure. So-called low floor–high ceiling tasks, where low floor

indicates tasks that are not so difficult to start with and high ceiling indicates tasks that may trigger other kinds of solutions on a higher cognitive level. Tasks that fulfil those "criteria" may help students at different levels with different strategies. I had to beware of what Ponte and Quaresma (2016) are arguing for, which is that mathematical tasks that demand explanations and analysing solutions and strategies are challenging since the students don't have strategies to solve the problems.

I had to facilitate activity and get my pupil more motivated, where Bruner means we should have a focus on the child as an active problem solver (Manger et al., 2013). Moreover, two of the four major themes Bruner is emphasizing are motivation and structured learning. Structured learning can be understood as the basic foundation of the topic, where the focus in lecturing should be understanding and not memorizing (Solerød 2012). I will also highlight Skemp (1976), who emphasized the importance of relational understanding instead of instrumental thinking. I had to let my pupil develop a deeper understanding, especially in algebra, where he had missed some major components in his former schooling.

In the article by Berggren et al. (2019), it emerges that to increase students' academic performance, there has been an increasing interest in different strategies in the classroom. An important factor in the students' achievements is the teacher-student relationship (Hattie, 2009). "The teacher-student relationship that involves trust, compassion, caring and attention which are key elements in creating a positive educational climate" (Berggren et al., 2019). In my case, I had to let him feel more care and attention. Like Philosopher William James says, to have success as a teacher, it depends on two things. The ability to "meet the student directly*"* and for the second, the ability of "pedagogical judgement" (Løvlie, 2004).

It was important for me to see him and for the class to have mathematical discussions with each other, where they could show mathematical competence. My first priority was one of the eight elements Niss and Jensen (2002) call communication competence and adaptive reasoning as the five strands of proficiency by Kilpatrick et al. (2001).

In the mathematical discussions, it was important to be aware of that I did not have any preferred solutions or leading questions, what Herbal, Eisenmann & Breyfogle (2005) call *funnelling*. But according to Newman (1990), this form of teaching is also fruitful if the problems are complex and sometimes also productive. I experienced the importance of asking questions, for instance, the words "why" and "how". Schoenfeld (1992) also shows that repeating questions may trigger the student's self-reflection and thinking, and afterwards, the student will ask themselves questions. I believe that to facilitate Dialogic communication, the focus should be

on exchanging ideas and strategies. According to Truxaw and DeFranco (2008), a more inductive classroom will help students construct and articulate their mathematical meanings.

This leads me to a study by Naalsund (2012), where she interviewed pupils related to the tasks in the international TIMMS test. The results showed that the Norwegian results are relatively weak in algebra, (Bergem et al., 2016). The main results from Naalsund (2012) showed that the pupils understood the methods in mathematics they used and the argumentation, but she got very few explanations as to why they solved the task in that way. Most of the students said, "I just followed a rule". From her study, we can see that formal procedures are used in a highly algorithmic manner and not so much for a deeper understanding.

I had a strong feeling that my pupil's low self-confidence and motivation in mathematics were a combination of the people around him and himself. I still believed my focus was to keep up our plan. I could not fall back and relax; I had to improve and use my lectures to strengthen his motivation for mathematics. In my case, and according to Buber, we must respect the other person. To meet another person in this manner requires an ethical attitude by showing openness, respect, trust, and responsibility (Botnen et al., 2011). I was also aware that it was not my intention to form him in any way. Like Skjervheim (1996) says, "We cannot form students as if they were made of chrome and metal like a subject-object relationship".

Studies by R. J. Rosenthal and L. Jacobson (1972) and the so-called Pygmalion Effect showed that if teachers believe that their pupil is a high performer or a late bloomer, they are nicer to them, teach more material to them, and give more differentiated feedback when they get the wrong answer. The study also showed that the kids started to think of themselves differently, and in the end, they were transformed and performed significantly better.

Conversely, if the teachers think the pupil is a low performer, they don't want to teach them very much and are willing to accept a low-quality response. Even this project was controversial. I think teachers must keep in mind that perhaps some pupils are thinking that, "My teacher does not challenge me enough relative to others in the class". In my case, I am very aware of saying that my pupil's former teachers were not clever enough to follow up on him and meet him with higher expectations in mathematics, but I can't categorically reject those thoughts.

At the University

When teaching the preschool teachers at the university, I try to share my experience from my own praxis as a teacher in junior high and high school.

I must admit that I feel there are expectations from the students that give me the same feeling I got with my pupil in my story. There are many skills that we must prepare the students for. Like Shulman (1986) is talking about, they need specific skills and PCK (pedagogical content knowledge). It is not enough to learn know-how, like techne; they also need phronesis to get the practical wisdom in the classroom.

Since the students need to learn more mathematics and, at the same time, didactics, it must be a mix of both parts. The challenge is, when I am lecturing on a difficult topic, how much should I teach before I let the student group try to process and understand the topics by themselves? If I teach too much, there is the possibility that some of my students are taking after me, and we are reproducing too many teacher-controlled lessons.

In my practice, I try as much as possible to activate the students within the resources I have. For example, I use vertical surfaces where they can discuss mathematical problems and sketch up and discuss different solutions for the rest of the students in the group. Some of this inspiration has come from Liljedahl (2020). By organizing them into groups, it has been easier to simulate situations that can arise in the classroom, some of which are from my own background. To illustrate, I use situations where there are no obvious solutions or answers and where the students must use their practical wisdom.

I use problem-based learning, where the students must work in groups and simulate different kinds of tasks and topics from the syllabus. One of the goals is to let them show different types of mathematical competence. For example, I divide the students into groups of four, where two should act as students and two should act as teachers. Then they must simulate how to teach fractions, with the "pupils" following up with questions for the "teachers". The students have enjoyed this form of working, and I think this has been fruitful since "both sides" need to articulate their meanings. In this setting, we have been arguing that we need different tools to help the pupils navigate the mathematical landscape.

The importance of asking open questions is a major part of my lectures at the university and in problem-based learning. I regularly tell the story of what Sokrates demonstrated for Menon (Løkke, 2014). Sokrates wanted to show that all learning is not remembering. By asking open questions to an arbitrary poor boy from the street, Sokrates managed to build up the boy's understanding of the Pythagorean theme by just asking questions.

After telling the story about the street boy, it has been natural to discuss what potential every pupil has. Since many are performing better at oral mathematical exams in an approximate period of two days and nights, a natural question would be, how is this possible? In that period,

the pupils have worked on different topics, and through most of the period, the teachers have been interlocutors. Still, with this in mind, we have discussed how we can organize our lessons better.

I hope that by telling my story, there will be a great deal of potential and willingness to organize and customize our mathematics lectures differently. Our long experience, lack of experience, rigid plans, and the syllabus may obscure new ways of thinking. But "what is best for the pupil?" For me, it started with a few words from a pupil with low self-efficacy in mathematics. He really made me start reflecting on my own role as a math teacher.

Hopefully, this will be fruitful for my students at the university and their pupils in the future.

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